New vacuum state and symmetry breaking in polariton system

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(February 1, 2008)

Abstract

The polariton system is studied by a concise approach using a simple model. A new ground state with negative energy is obtained and found to exhibit the symmetry breaking.

PACS numbers: 71.36.+c, 11.30.Qc

Key Words: Polariton, Symmetry Breaking

The nonclassical properties of light have become a fascinating subject of widespread investigations in recent years due to their importance in the research of laser and the design of new light sources. Some nonclassical states of light such as the squeezed states, the pair-coherent states, and the photon number states have been extensively studied both theoretically and experimentally [1–6]. Recently, efforts have been made to study the nonclassical behavior of light in model solid state systems [7,8]. When light falls on a solid-state material and interacts with the vibrating lattice, a photon-phonon complex can be formed, which is called a polariton. Ghoshal and Chatterjee [7,8] have considered two possible model polariton systems, which involve one mode of the photon field interacting with a single optical phonon, and solved them by a canonical transformation to the Hamiltonian. Their results show that both the phonon and photon subsystems can exhibit nonclassical behavior.

In this Letter, we report the ground-state properties of the polariton system. We adopted the model of Ghoshal and Chatterjee [8] for the polariton system, and used a concise approach [9], where the wave function is taken as the coherent-like form, to solve the Schrödinger equation exactly. We obtained a new ground state whose energy is lower than the ordinary vacuum one for the given model [8]. This new ground state can be regarded as the true vacuum and it exhibits the symmetry breaking.

Before we present our results, let us describe Ghoshal and Chatterjee's model and results briefly [8]. The Hamiltonian of the model of Ghoshal and Chatterjee is as follows [8]

$$H = \omega_a a^+ a + \omega_b b^+ b + \kappa(a^+ b^+ + ab + a^+ b + b^+ a) \tag{1}$$

where $a^+(a)$ is the creation(annihilation) operator for an optical phonon of frequency ω_a, $b^+(b)$ is the corresponding operator for the photon field, ω_b being the photon frequency and κ is the phonon-photon coupling strength. Using the following transformation

$$\begin{cases} a = A_1\alpha + A_2\alpha^+ + B_1\beta + B_2\beta^+ \\ b = B_3\alpha + B_4\alpha^+ + A_3\beta + A_4\beta^+ \end{cases} \tag{2}$$

and choosing suitable parameters $\{A_i,\, B_i\}$ $(i = 1,\, 2,\, 3,\, 4)$, the expression of Hamiltonian can be diagonalized as [8]

$$H = E_\alpha \alpha^+ \alpha + E_\beta \beta^+ \beta + E_0 \tag{3}$$

where E_α, E_β, and E_0 depend on ω_a, ω_b and κ [8].

We take the form of solution as

$$|A\rangle = e^{\alpha a^+ a^+ + \beta b^+ b^+ + \gamma a^+ b^+} |0\rangle \tag{4}$$

where α, β, and γ are parameters to be determined, $|0\rangle$ is the ordinary vacuum of the phonon and photon. If this state is really a solution of the model, it should obey the Schrödinger equation

$$H|A\rangle = E|A\rangle \tag{5}$$

Substituting Eq.(1) and Eq.(4) into Eq.(5), then comparing the terms of $|A\rangle$, $a^+a^+|A\rangle$, $b^+b^+|A\rangle$ and $a^+b^+|A\rangle$ respectively, we obtain

$$E = \kappa\gamma \tag{6}$$

$$\omega_a \alpha + \kappa\gamma + \kappa\gamma\alpha = 0 \tag{7}$$

$$\omega_b \beta + \kappa\gamma + \kappa\gamma\beta = 0 \tag{8}$$

$$\omega_a \gamma + \omega_b \gamma + \kappa + \kappa\beta + \kappa\alpha + \kappa(\alpha\beta + \gamma^2) = 0 \tag{9}$$

¿From these equations the equation satisfied by E is obtained

$$(E + \omega_a)(E + \omega_b) - \omega_a\omega_b + \frac{\kappa^2 \omega_a \omega_b}{(E + \omega_a)(E + \omega_b)} = 0 \tag{10}$$

Introducing

$$S = (E + \omega_a)(E + \omega_b) \tag{11}$$

Eq.(10) can be rewritten as

$$S^2 - \omega_a\omega_b S + \kappa^2 \omega_a \omega_b = 0 \tag{12}$$

We have

$$S=\frac{\omega_a\omega_b \pm \sqrt{\omega_a\omega_b(\omega_a\omega_b-4\kappa^2)}}{2} \tag{13}$$

Then four solutions are obtained

$$E=\frac{1}{2}[-(\omega_a+\omega_b)\pm\sqrt{\omega_a^2+\omega_b^2\pm 2\sqrt{\omega_a\omega_b(\omega_a\omega_b-4\kappa^2)}}] \tag{14}$$

if

$$\omega_a\omega_b > 4\kappa^2 \tag{15}$$

When we impose the normalization condition to these wavevectors, we have the following constraints for α, β and γ

$$\begin{cases} \alpha<\frac{1}{2} \\ \beta<\frac{1}{2} \\ \gamma<1 \end{cases} \tag{16}$$

With these constraints and the condition Eq.(15), only one solution is left, *i.e.*,

$$E=\frac{1}{2}[-(\omega_a+\omega_b)+\sqrt{\omega_a^2+\omega_b^2+2\sqrt{\omega_a\omega_b(\omega_a\omega_b-4\kappa^2)}}] \tag{17}$$

This energy is negative and lower than the ordinary vacuum energy. Therefore there is only one true vacuum. This ground state does not display the symmetry of the Hamiltonian under the gauge transformation

$$U=exp\{i(a^+a+b^+b)\} \tag{18}$$

Then we have a broken symmetry.

Let us discuss briefly the physical meanings of the new vacuum state. If its phonon frequency(ω_q) is approximately independent of the wavevector (q), the material can be regarded as the one with only one mode of phonon. We put this material into a cavity with one mode of radiation field. When the temperature gets lower and lower, the probability of the new vacuum increases larger and larger and we will find even the temperature approaches to the absolute zero, the average number of phonons arrives at a definite non-zero value.

This phenomenon is quite different from the one without the existence of the new vacuum state. If there would not be the new vacuum, the ordinary vacuum $|0\rangle$ would dominate in the probability distribution when the temperature approaches absolute zero, i.e., the phonon number in the material would tend to be zero.

This new vacuum state will have influence on some physical quantities which could be measured by the experiment. When the material is put in the cavity under enough low temperature there will be transitions from the exited states of polariton to the new vacuum state, and then some new lines in spectrum will be found. For the same reason, there will be new structures in the absorption spectrum. On the other hand, this new vacuum state may also have the contribution on the specific heat of the material. We hope these interesting results can stimulate experiments to study the polariton system.

From the discussions above, we can conclude here, that in the model polariton system we studied, there is a true vacuum state, which has really lower energy than $|0\rangle$, provided the condition Eq.(15) is satisfied. This result is interesting because the true ground state or the symmetry breaking is obtained without any assumption and approximation in this case.

ACKNOWLEDGMENTS

This work was supported by the National Climb Project "Nonlinear Science" of China. We would like to thank Professor S.L. Wan for helpful discussions.

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